

**The Spatial Distribution of Population in 35 World Cities: The Role of Markets,  
Planning, and Topography**

By

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## Introduction

Urban economists have studied the spatial distribution of population intensively since the pioneering work of Alonso (1964), Muth (1969) and Mills (1972). Of course this work has a longer history, traceable at least back to von Thunen (1826), including studies by other social scientists such as Burgess (1925), Hoyt (1959) and Clark (1951). In addition to population distribution, there is a related empirical literature on the distribution of real estate prices (e.g. Follain and Malpezzi 1981) and the distribution of wages and incomes over space (Eberts 1981, Madden 1985). Much of this literature is ably surveyed by McDonald (1989). Broad reviews of the theoretical models behind this empirical work can be found in, for example, Wheaton (1979) Straszheim (1987) and Arnott *et al.* (1998).

## The Measurement of Urban Form

### *Population Density Gradients*

The measure of city form that has been most often studied by urban economists is the population density gradient from a negative exponential function, often associated with the pioneering work of Alonso, Muth and Mills, but actually first popularized among urban scholars by the geographer Colin Clark.<sup>1</sup> More specifically, the population density of a city is hypothesized to follow:

$$D(u) = D_0 e^{-\gamma u + e}$$

where  $D$  is population density at distance  $u$  from the center of a city;  $D_0$  is the density at the center;  $e$  is the base of natural logarithms;  $\gamma$  is "the gradient," or the rate at which density falls from the center. The final error term,  $e$ , is included when the formulation is stochastic.

Among the other attractive properties of this measure, density is characterized by two parameters, with a particular emphasis on  $\gamma$ , which simplifies second stage analysis. The function is easily estimable with OLS regression by taking logs:

$$\ln D(u) = \ln D_0 - \gamma u + e$$

which can then be readily estimated with, say, density data from Census tracts, once distance of each tract from the central business district (CBD) is measured.

The exponential density function also has the virtue of being derivable from a simple model of a city, albeit one with several restrictive assumptions, e.g. a monocentric city, constant returns Cobb-Douglas production functions for housing, consumers with identical tastes and incomes, and unit price elasticity of demand for housing.

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<sup>1</sup> McDonald, in his excellent (1989) review, points out that Stewart (1947) apparently first fit the negative exponential form described here, but notes that it was Clark (1951) that popularized the form among other urban scholars.

As is well known, the standard urban model of Alonso, Muth and Mills predicts that population density gradients will fall in absolute value as incomes rise, the city grows, and transport costs fall. Extensions to the model permit gradients to change with location-specific amenities as well (Follain and Malpezzi 1981).

The negative exponential function often fits the data rather well, for such a simple function in a world of complex cities. Sometimes it does not fit well, as we will confirm. Many authors have experimented with more flexible forms, such as power terms in distance on the right hand side.

The world is divided up into two kinds of people: those who find the simple form informative and useful, despite its shortcomings (e.g. Muth 1985), and those who believe these shortcomings too serious to set aside (e.g. Richardson 1988).<sup>2</sup> In fact, given the predicted flattening of population density gradients as cities grow and economies develop, it can be argued that the monocentric model on which it rests contains the seeds of its own destruction; and that a gradual deterioration of the fit of the model is itself consistent with the underlying model.

### *Other Measures*

In addition to the traditional gradient measure, many other measures of urban form have been put forward and studied. The simplest, of course, is the average density of the city or metropolitan area. Others include measures such as the weighted average of straight line or rectangular distances from one set of points in a city to another, or functions based on densities other than the negative exponential, such as the normal density (Ingram 1971; Pirie 1979; Allen et al. 1993). However we would like to consider an alternative measure of urban form, which we term a compactness measure.

### *Compactness*

From experience we know that cities come in many different shapes. Cities' shape can be defined by three variables: the surface of the built-up area, the shape of the built-up area and the way the population density is distributed within this same built-up area.

We can thus represent a city as a 3-dimensional object. The shape of the built up area will be represented in the xy plane, and density will be represented in the z plane. We thus obtain a solid whose geometric properties can be analyzed. Such a solid has a center of gravity (COG) which is the point to which the sum of distance from all other points of the shape is the shortest.

All else equal, a city shape which decreases the distance between people's residences and the main place of work and consumption will be more favorable to the functioning of labor and consumer markets. For a given built-up area, the shorter the average distance per person to the main place of work or to the main commercial areas, the better would be the performance of the city shape.

Traditionally, planners and urban economists consider that a city is either monocentric or polycentric, depending on the location of the main employment and retail centers. In reality

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<sup>2</sup> The world is also divided up into people who divide the world into two kinds of people, and people who don't, but that's another paper.

no city is purely monocentric or purely polycentric. Cities have only degrees of monocentrism and polycentrism. In a realistic classification of cities there would be a continuum between very monocentric toward very polycentric cities, with most cities located in between.

In general, polycentric cities do not contain two or 3 centers but a large number of small centers. These centers do not act as CBD for the surrounding areas, they are not the center of minicities. Because their accessibility from the rest of the metropolitan area, they are points of condensation of employment and commerce. Their catchment area is in fact the entire metropolitan area. If it was not, large polycentric cities would have the productivity linked to the scale of their component elements. In a very polycentric city, employment and commerce are widely distributed among many small centers, and that the trips they generate are widely distributed across the metropolitan area. By contrast a theoretically pure monocentric city generates only trips along its radius.

The challenge is to develop a compactness index which would be applicable to a monocentric and to a polycentric city. In a monocentric city the main place of work and commerce would be the CBD. In a polycentric city the center of gravity of the shape would be the closest point to all centers of employment and commerce. The measure of the average distance per person to the CBD – in case of a monocentric city – or to the center of gravity of the population – in case of a polycentric city – provides a good measurement of the performance of the city shape.

Our compactness index, rho, is the ratio between the average distance per person to the CBD, and the average distance to the center of gravity of a cylindrical city whose circular base would be equal to the built-up area, and whose height will be the average population density:

$$r = \frac{\sum_i d_i w_i}{C}$$

where rho is the index, d is the distance of the ith tract from the CBD, weighted by the tract's share of the city's population, w; and C is the similar, hypothetical calculation for a cylindrical city of equivalent population and built up area. A city of area X for which the average distance per person to the CBD is equal to the average distance to the central axis of a cylinder which base is equal to X would have a compactness index of 1. See Figure 1.

Of course the denominator, C, is merely a baseline against which to compare the actual compactness of the city. We are not arguing that cylindrical cities are in some sense optimal, merely that some cities will be more compact than this baseline (have a lower value of rho), and some will be less compact (have a higher value of rho).

In the majority of cities the CBD and the center of gravity of the population coincide. This is the case for Beijing, London, Moscow, New York, Paris, Shanghai, and many other cities. In others the CBD is quite distant from the center of gravity. We would expect that cities in which the CBD is eccentric (far away from the center of gravity) would be mostly polycentric. In a polycentric city the location of CBD is not very important as more trips are generated to and from many other sub-centers. A number of the cities we have studied which have an eccentric CBD are predictably dominantly polycentric (Houston, San

Francisco, Los Angeles, Rio de Janeiro) but others are dominantly monocentric (Bombay, Curitiba, Cracow). The compactness index we propose here – based on distance to the CBD – is a potentially a good indicator of sprawl for dominantly monocentric cities, and for polycentric cities in which the center of gravity and the CBD coincide. The compactness index – as calculated in this paper – might have less significance for polycentric cities where the CBD is eccentric.

## The Determinants of Urban Form

Many papers are devoted to measurement and specification issues, e.g. what form the so-called density gradient should take. But perhaps the most interesting literature is that which explains *variation* in patterns of population density among cities.

The well-known "standard urban model" of Alonso (1964), Muth (1969) and Mills (1972)<sup>3</sup> postulates a representative consumer who maximizes utility, a function of housing (H) and a unit priced numeraire nonhousing good, subject to a budget constraint that explicitly includes commuting costs as well as the prices of housing (P) and nonhousing (1). It is easy to show that equilibrium requires that change in commuting costs from a movement towards or away from a CBD or other employment node equals the change in rent from such a movement. For such a representative consumer:

$$\partial u \cdot t = -\partial P(u) \cdot H(u)$$

where  $u$  is distance from the CBD and  $t$  is the cost of transport. This equilibrium condition can be rearranged to show the shape of the housing price function:

$$\frac{\partial P(u)}{\partial u} = -\frac{t}{H(u)}$$

Now consider two consumers, one rich and one poor. Assume H is a normal good. If (for the moment),  $t$  is the same for both consumers but H is bigger for the rich (at every  $u$ ), the rich bid rent function will be flatter. The rich will live in the suburbs and the poor in the center. Even if  $t$  also increases with income (as is more realistic), as long as increases in H are "large" relative to increases in  $t$ , this result holds. Also, as incomes rise generally, the envelope of all such bid rents will flatten. Also, clearly, as transport costs fall, bid rents will flatten.

The standard urban model has a competitor, which is sometimes called the "Blight Flight" model (Follain and Malpezzi 1981). As presented in the U.S. literature, the Blight Flight Model has a negative tone. People have left the cities not because they preferred suburban living a la the standard model, but because the cities themselves have become less desirable places to live. As U.S. cities became more and more the habitat of low-income households and black households, the argument goes, housing and neighborhood quality declined and white middle-to-upper income households flew to the suburbs.

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<sup>3</sup> These three references are among the classics in the field. Among many recent treatments and extensions, see Fujita (1989), Turnbull (1995) and Arnott, Anas and Small (1998).

While "Blight Flight" explanations focus on negative amenities such as crime and fiscal stress, the models are easily generalizable to positive amenities such as high quality schools. Blight Flight can be generalized and formalized by adding a vector of localized amenities (and disamenities) to the standard urban model above. See, for example, Li and Brown (1980), Diamond and Tolley (1982), and many subsequent applications.

Thus the two models - "standard urban" and "amenities" - are not mutually exclusive. Which theory has more explanatory power? In this research project we aim to model the determinants of population distribution within cities, including those which come out of the so-called standard urban economics model of Alonso, Muth and Mills, e.g. incomes and transport costs; and the so-called amenity/disamenity or "Blight Flight" models. We also investigate the effect government action, e.g. planning and land use regulation, and public real estate investment, has on the form of the city. We also intend to examine the effect of natural geographical constraint.

In this draft, we have focused mainly on income and transport and, in a rough way, government intervention. We are currently collecting more data to enable additional work on the effects of amenities and natural constraint.

In some respects Mills and Price (1984) is a model for what we're trying to do. In the first stage of Mills and Price's paper, they estimated the population density gradient for each of about 50 metropolitan areas. These results were then used as the dependent variable in a second stage, where the gradients were regressed on a number of possible determinants, some suggested by the standard urban model of Muth, Mills, *et al.*, and some by so-called "Blight Flight" or disamenities models of the city.

Key results from the Mills and Price second stage regression of urban population density gradients and employment density gradients on their determinants included the following:

- (1) The most important determinant of gradients in a given year are gradients in previous years.
- (2) Cities with higher average incomes may have flatter population gradients but we are not sure (coefficient has right sign but t-statistic is small).
- (3) Larger cities have flatter gradients.
- (4) A higher percentage of minority households in the central city flattens the gradient.
- (5) Relative wages had the wrong sign. Higher crime in the central city appeared to *steepen* the gradient. Better education in the central city appeared to *flatten* the gradient. These results were difficult to explain, at best.

Specific results were less robust than may be desirable. For example, a somewhat similar exercise by Follain and Malpezzi (but using house prices rather than population density) found

some quite different results, e.g. opposite signs on race and poverty variables from Mills and Price's. Thus, extending the Mills and Price model is one goal of this paper.<sup>4</sup>

The majority of empirical studies in this vein (published in English) have been about U.S. metropolitan areas. Even casual observation of cities within the U.S. versus cities in the rest of the world suggests a larger view could be fruitful. Informal empiricism tells us the variation in both the distribution of population, and in many determinants, will be greater across countries than within the U.S. Cross country analysis is interesting on its own terms, of course, but it may also prove to be more robust than analysis of data from a single country. Increasing variance in the dependent and independent variables should increase the power of tests.

Of course a number of studies have examined the distribution of population in one or a few markets outside the U.S. For example, Bertaud and Renaud (1997) have examined Russia (see Mozolin (1994) for a related study of housing prices); Asabere et al. (1982, 1983), for analysis of Ghanaian cities; DiPasquale (1996) has studied Chile; Glickman (1979) Japan, and Mills and Song (1979) Korea. But except for Mills and Tan (1980), careful comparisons of such outcomes across countries are hard to find.

Mills and Tan's survey of international studies of population density is, in many respects, the closest to this paper we've found in the literature so far. Those authors make a number of careful comparisons among a wide range of studies, most using the negative exponential model, e.g. Brush (1968), Ingram and Carroll (1980), Mills and Ohta (1976), and Mills and Song (1979). Mills and Tan relate flattening gradients to rising incomes and growing cities, but in a somewhat qualitative, informal way. That is, Mills and Tan generally presented tabular evidence, e.g. of average density gradients by city size and by country (and hence by GDP per capita). They presented evidence that population density gradients fall over time, worldwide; and that this is further related to growth in incomes and the size of cities. Given the wide range of data sources and estimation procedures followed by the studies that form the base of their comparison, Mills and Tan were careful to make mainly qualitative comparisons.

In our study, we have the advantage of comparable data collected and analyzed for over 30 large metropolitan areas around the world.

## **Data**

The first contribution of this study, then, is to consistently estimate a series of population density measures (the traditional gradient, and compactness) for 35 major metropolitan areas around the world. We then model their determinants. The cities for which we have developed data include:<sup>5</sup>

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<sup>4</sup>This paper complements ongoing work using domestic data. Malpezzi and Kung (1997) presented preliminary results from an extension of the Mills and Price framework to recent U.S. metropolitan data. Malpezzi and Kung's empirical work is being extensively revised, and a revised version will be available shortly.

<sup>5</sup> The "we" in this sentence is editorial. Credit for the painstaking data collection - by far the bulk of the work undertaken for this paper - is due the first author, Alain Bertaud. Analytic duties were shared. (S.M.)



Abidjan	Los Angeles
Ahmedabad	Moscow
Bangkok	New York
Beijing	Paris
Berlin	Portland
Bombay	Rio de Janeiro
Capetown	San Francisco
Chicago	Seoul
Cracow	Shanghai
Curitiba	Singapore
Guangzhou	St Petersburg
Hong Kong	Tianjin
Houston	Tunis
Hyderabad	Washington
Jakarta	Yerivan
Johannesburg	London

The best way to begin the analysis of differences among cities is to examine city-specific charts of density by distance. These are contained in the Data Appendix. Dots on the charts indicate average density of built-up areas at each 1 km annulus from the center of the city. The solid line indicates the exponential density gradient. Note in particular the inversion of the classic declining population density in cities with heavy-handed planning legacies (Moscow, Seoul), and (the ultimate perversion of planning) apartheid cities (Johannesburg and Capetown).

### **Simple Models of Determinants**

One way we can gain insight into decentralization is to model and empirically estimate the determinants of population gradients. Here we note again, as above, that although many studies have been carried out using the simple negative exponential form, the form has been criticized on several grounds. A large literature exists which models a city in a one way or another as a collection of many centers or nodes, often using simulation approaches rather than analytics (see Arnott and Anas 1994, Kain and Apgar 1979, Ingram 1979, Richardson 1988, and Yinger 1994 among many others). Careful empirical tests of the negative exponential form in one or a few cities usually (and unsurprisingly) find that more flexible forms fit cities better. A few papers have examined simple (two or three parameter) alternatives to the negative exponential, such as Bradford and Kelijian (1973) and Mills (1992).

While these critical studies make valid and important points, we believe the simple negative exponential model is still useful for comparative work. The best analysis of any individual city is undoubtedly undertaken using more flexible forms or even a simulation approach; but it is expensive and difficult to undertake such analyses in a consistent way for many cities for comparative work of the sort attempted here, and even if the required resources were available, it is hard to characterize results from such models in a parsimonious way. Parsimony is clearly required if we are to model a manageable number of city-specific parameters in a second stage. We also note that estimates of the fit of these simple models are developed as we go, and these fit statistics themselves contain valuable information about decentralization which can be

exploited in future empirical work.<sup>6</sup> In sum, we view comparative work based on simple models such as the negative exponential as complements to, rather than as substitutes for, more detailed and "realistic" approaches to one or a few cities.

The thrust of this paper is therefore straightforward: in stage one we estimate  $\delta$  and  $\rho$  separately for each of a number of metropolitan areas. In stage two we examine the determinants of each in turn, using in addition to the endogenous variables a set of exogenous variables which can represent various elements of the SUE and Blight Flight models. If, for example, incomes rise, or transport costs fall, the SUE model predicts declining (flatter) population gradients. The Blight Flight/amenities model suggests increases in central city crime or poverty will have similar effects. But at the same time, as demand shifts in favor of one good over another, e.g., suburban housing versus central-city housing, the price of the good experiencing increased demand will increase relative to the other. In principle the market can be equilibrated by population shifts, or price changes, or some combination of the two. By studying whether variations in density gradients and the prices of city vs. suburban housing can be explained by variables suggested by the two theories, we gain insights into the validity and relative predictive power of the theories. Within theories, we gain insight into which determinants matter most -- for example, assuming the SUE model "works," which has more predictive power, changes in income, or changes in transport cost? Which amenities or disamenities have the strongest locational effect?

The answers to questions like these matter. Many public policies -- enterprise zones, fair housing enforcement, urban renewal -- have been put in place at one time or another to address the decline of American central cities, and new (and recycled) policies are currently under consideration. Many other policies -- transport policies, welfare and educational policies, tax and financial policies -- profoundly affect the form of the city and the welfare of its inhabitants that can be well understood only if we understand the dynamics of location.

The "natural" way to estimate  $\delta$  for each metropolitan area is to compute population densities at a number of locations within the metropolitan area then estimate a log-linear regression of these densities against the distance of each location from the center,  $u$ :

$$\ln D_{ij} = \ln D_{0i} - \delta_i u_{ij} + \epsilon_{ij}$$

where  $i$  indexes the metropolitan area and  $j$  indexes intrametropolitan locations;  $\epsilon$  is an error term.

The compactness indicator,  $\rho$ , is also calculated separately for each city, following equation (3), above.

In the second stage, the MSA is the unit of observation. Using the notation introduced above, we model the determinants of population gradients and compactness:

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<sup>6</sup>We have not modeled these fits in this first draft. We will also investigate quadratic and other forms in future drafts.

$$d, \rho = f(Y, t, T, A, G)$$

where in addition to the endogenous variables defined above  $Y$  is income (proxied here by GDP Per Capita);  $t$  is a vector representing transport infrastructure;  $T$  is a set of tax and fiscal variables;  $A$  is a vector of locational amenities (including spatial measures of poverty and race, and locational specific public goods such as school quality);  $G$  are variables measuring geographical constraints; and  $R$  are measures of regulation. Locational subscripts ( $i$ ) are omitted.

## Data

In these first results, we have the dependent variables, and mainly measures of  $Y$ ,  $t$  and  $R$ . Indicators of  $T$ ,  $A$  and  $G$  are under development.

The compactness and density gradient indexes are constructed from data collected city by city by the first author, on field trips to each city. Generally the data are from Census and other official sources. Data are collected and analyzed for built-up areas only, i.e. these are "net" results. Areas such as large parks and bodies of water, or undeveloped greenbelts, are omitted.

Among independent variables, most (city population, GDP per capita, motor vehicles per 1,000 population, etc.) are self explanatory. These are generally taken from World Bank sources. One variable which is not is the dummy for "excessive regulation."

A theme of much of our previous work, separately and together, is the role regulation plays in housing and real estate markets, urban form, and consumer welfare. Regulation *per se* is, of course, neither good nor bad. What matters is the cost and benefit of specific regulations under specific market conditions.

That said, in work such as Angel and Mayo (1996), Bertaud (1989, 1992 a, b, 1997), Malpezzi (1990), Malpezzi and Ball (1993) and World Bank (1993) we and others find evidence that many cities in many countries systematically over-regulate housing and real estate markets. In several of those papers various indices of regulation have been developed. Unfortunately, the overlap between cities with independent regulatory measures and our set of cities is insufficient for estimation. We therefore created a crude 0-1 dummy variable for markets that, in our opinion, are excessively regulated. In this context, we don't merely mean planning, zoning or building codes that are a little restrictive, as in say New York or London or San Francisco. Rather, we mean a regulatory regime that ignores or violently contradicts market forces, such as found in Moscow or Seoul or Johannesburg. If anything, many of the "moderately" regulated cities with values of zero, such as London or New York, can be argued to be restrictive in their own right. The difference is a matter of degree.

We point out that since this index is subjective, and constructed by the same authors that collected the data for and constructed the dependent variables, the possibility of bias, or if you

prefer, endogeneity in the regulatory dummy, is real. We've attempted to construct the regulatory dummy without reference to the density patterns, but of course we can't be sure how successful we are at blocking out this knowledge. In future drafts we hope to make use of additional third-party measures of real estate regulation, such as those undertaken by the UN Housing and Urban Development Indicators Programme (Angel and Mayo 1996; see also the United Nations Centre for Human Settlements (Habitat) website).

### **First Stage Results: Gradients, and Compactness**

Table 1 presents the gradient estimates, compactness indexes, and several other key variables, for each city. Cities are listed alphabetically. For New York, San Francisco and Seoul, separate estimates were undertaken for the city (corresponding roughly to a metropolitan area), and to a broader region (roughly corresponding to a U.S. CMSA). In this paper we've dropped the broader regions from subsequent analysis.

In the event, most cities have negative population density gradients, as predicted by the standard urban model. Capetown, Moscow, and Seoul have inverted gradients, i.e. population density increases with distance from the center, at least over some relevant range.

Figures 1 and 2 present the gradient and compactness measures by GDP per capita. On average, the gradient falls with GDP, as the standard urban model predicts (Figure 2). This is true even if the influential observation of Guangzhou is omitted from the dataset. It is also apparent that significant variation exists around means conditional on income; for example, Bombay, though one of the lower income cities in the sample, has a practically flat gradient. Singapore has a steep gradient for such a high-income city. Heteroskedasticity is also apparent in Figure 1.

Compactness varies less systematically across income (Figure 3). Bombay is a clear and very influential outlier in compactness.

The gradient also flattens as cities grow in population, although again there is significant variation around trend, and changing variance (Figure 4). There is some apparent heteroskedasticity in city size for the compactness measure as well (Figure 5).

Cities with steeper gradients are generally more compact. That can be true even when they have a larger overall "footprint." Figure 6 illustrates with a three dimensional representation of the population densities of built up areas of Paris and Moscow, two cities of about 8 million, both with fairly centralized employment patterns for cities of their size. Now, Paris very broadly exhibits the classic gradient from the standard urban model, despite strong planning controls, height limitations in the center, and the promotion of new town development on the periphery. Moscow has developed with a much smaller footprint, and very dense housing on the periphery of the city. But despite the *apparent* compactness from a smaller footprint, Moscow is actually much less compact than Paris. The dense developments of Moscow's periphery put enormous demands on commuting and on transport infrastructure.

Table 2 presents additional summary statistics from Table 1. These are presented for all cities, and by the regulatory dummy.

As measured, the larger the gradient *in absolute value*, the steeper the drop-off of population density. Thus, heavily regulated cities have flatter gradients, when means or medians are compared. The larger the compactness index, the less compact the city. Heavily regulated cities are less compact, on average or "on median," than moderately regulated cities.

## **Second Stage Results: Determinants of Gradients, and Compactness**

Table 3 presents the results of OLS regression estimates of the determinants of density gradients and of our compactness measure. The independent variables have been discussed above, with the exception of the dummies for Seoul and Bombay.

In preliminary estimation, we found that in all variations of the density gradient models, Seoul was an outlier and an *extremely* influential observation, in the Belsley, Kuh and Welsch (1980) sense. The same was true for Bombay in the compactness regressions. We decided to "dummy out" these influential observations, based on the value of several diagnostic tests.

Of omitted variables, one we'd most want is a measure of the centrality of employment. A city with dispersed population can still be efficient, if employment is also dispersed and significant agglomeration economies are not foregone by this dispersal. However, we note that the cities with the inverted gradients (most decentralized population) are Capetown, Moscow and Seoul, and all three have high proportions of their employment in the central city (see World Bank 1993 b, and Bertaud and Renaud 1997).

In the future, we would also like to add better measures of geography (natural constraint) and, if possible, variables capturing fiscal and localized amenity differences.

In the event, our preliminary gradient regressions perform quite well. Fit is excellent, for such international cross-city comparisons. Results for other variables are insensitive to the inclusion of the regulatory dummy, which is comforting. Gradients increase (flatten) with the log of income, and the log of city population, as predicted by the standard urban model. The higher the proportion of motor vehicles to a country's population, *ceteris paribus*, the flatter the gradient. The countries with "excessive" regulation also have flatter *ceteris paribus* gradients, even after dummifying out Seoul.

The compactness regression results are more mixed. Fit is acceptable, but compactness *per se*, at least as we've measured it, is not related to income, population, or the presence of automobiles. It is related to regulation, at least as measured, and Bombay is clearly an outlier and influential. In other regressions, no shown, we found that dropping Bombay, and leaving Bombay in the sample and dropping the dummy variable, had no qualitative effect in that income, population and transport remain largely insignificant.

## **Conclusions**

In this paper we have calculated, on a consistent basis, population density gradients for over thirty large cities in some twenty countries. We have also constructed an alternative measure of city population compactness.

In a second stage model we find that density gradients flatten with income, with city population, and with falling transportation costs, as the standard urban model predicts. We also find that cities with extremely repressive urban regulations, as in South Africa, Korea and Russia, have flatter (sometimes inverted) population density gradients. However, we would particularly like to improve our measurement of the urban policy environment in future drafts.

We are so far somewhat less successful at explaining variation in our compactness index. The regulatory environment is the most consistent systematic determinant.

Future drafts will include more careful consideration of natural constraint (geography) and amenities; will examine the relationship between compactness and traditional density gradients more carefully; and will attempt to better capture elements of the regulatory environment and other public policies affecting housing and real estate development.

## References

- Allen, W. Bruce, Dong Liu and Scott Singer. Accessibility Measures of U.S. Metropolitan Areas. *Transportation Research -- B*, 27B(6), 1993, pp. 439-49.
- Alonso, William. *Location and Land Use*. Harvard University Press, 1964.
- Angel, Shlomo and Stephen K. Mayo. Enabling Policies and Their Effects on Housing Sector Performance: A Global Comparison. Paper presented to the Habitat II Conference, Istanbul, Turkey, June 1996.
- Arnott, Richard, Alex Anas and Kenneth Small. The Welfare Economics of Urban Structure. Working Paper, 1998.
- Asabere, Paul K. and K. Owusu-Banahene. Population Density Function for Ghanaian (African) Cities: An Empirical Note. *Journal of Urban Economics*, 14, 1983, pp. 370-9.
- Asabere, Paul K., C.F. Sirmans and Peter F. Colwell. The Intensity of Residential Land Use in Accra, Ghana. *Journal of Urban Economics*, 11, 1982, pp. 190-8.
- Belsley, David A., Edwin Kuh and Roy E. Welsch. *Regression Diagnostics: Identifying Influential Data and Sources of Collinearity*. Wiley, 1980.
- Bertaud, Alain. The Regulatory Environment for Urban Land in Indonesia: Constraints Imposed Upon the Poor. World Bank, Processed, 1989.
- Bertaud, Alain. China: Urban Land Use Issues. World Bank, Processed, 1992.
- Bertaud, Alain. Russia: Urban Land Use Issues. World Bank, Processed, 1992.
- Bertaud, Alain. Measuring Constraints on Land Supply: The Case of Hong Kong. World Bank, Processed, 1997.
- Bertaud, Alain. The Spatial Distribution of Population in Cracow: A City's Structure Under the Conflicting Influences of Land Markets, Zoning Regulations and its Socialist Past. Paper presented to the Lincoln Institute of Land Policy Conference on Land Prices, Land Information Systems, and the Market for Land Information, Cambridge, Mass., November 1997.
- Bertaud, Alain and Bertrand Renaud. Socialist Cities Without Land Markets. *Journal of Urban Economics*, 41, 1997, pp. 137-51.
- Bertaud, Alain and Stephen Malpezzi. Measuring the Costs and Benefits of Urban Land Use Regulation: A Simple Model with an Application to Malaysia. University of Wisconsin, Center for Urban Land Economics Research, 1998.
- Bradford, David and Harry Kelejian. An Econometric Model of the Flight to the Suburbs. *Journal of Political Economy*, 81(3), May June 1973, pp. 566-89.

Brush, J.E. Spatial Patterns of Populations in Indian Cities. *Geographical Review*, 58, 1968, pp. 362-91.

Burgess, Ernest W. The Growth of a City: An Introduction to a Research Project. *Publications, American Sociological Society*, 18(2), 1925, pp. 85-97.

Clark, Colin. Urban Population Densities. *Journal of the Royal Statistical Society*, 114, 1951 pp. 375-86.

Clark, Colin. Land Use in Urban Areas. Chapter 9 of *Population Growth and Land Use*. London: Macmillan, 1967.

Cropper, Maureen L. and Peter L. Gordon. Wasteful Commuting: A Reexamination. *Journal of Urban Economics*, 29, 1991, pp. 2-13.

Cummings, Jean and Denise DiPasquale. The Spatial Implication of Housing Policy in Chile. Working Paper, 1994.

Diamond, Douglas B. Jr. and George Tolley. *The Economics of Urban Amenities*. Academic Press, 1982.

Eberts, Randall W. An Empirical Evaluation of Intraurban Wage Gradients. *Journal of Urban Economics*, July 1981, pp. 50-60.

Follain, James R. and Stephen Malpezzi. The Flight to the Suburbs: Insight from an Analysis of Central City versus Suburban Housing Costs. *Journal of Urban Economics*, May 1981.

Fujita, Masahisa. *Urban Economic Theory*. Cambridge University Press, 1989.

Glickman, Norman J. *The Growth and Management of the Japanese Urban System*. Academic Press, 1979.

Hoyt, Homer. *The Structure and Growth of Residential Neighborhoods in American Cities*. FHA, 1939. Selections reprinted in H.M. Mayer and C.F. Kohn (eds.), *Readings in Urban Geography*. Chicago, 1959.

Hoyt, Homer. *Where the Rich and the Poor People Live*. Urban Land Institute, 1966.

Ihlanfeldt, Keith R. Intra-Urban Wage Gradients: Evidence by Race, Gender, Occupation Class and Sector. *Journal of Urban Economics*, 32, 1992, pp. 70-91.

Ingram, D.R. The Concept of Accessibility: A Search for an Operational Form. *Regional Studies*, 5, 1971, pp. 101-7.

Ingram, Gregory and Alan Carroll. The Spatial Structure of Latin American Cities. *Journal of Urban Economics*, 9(2), 1981, pp. 257-73.

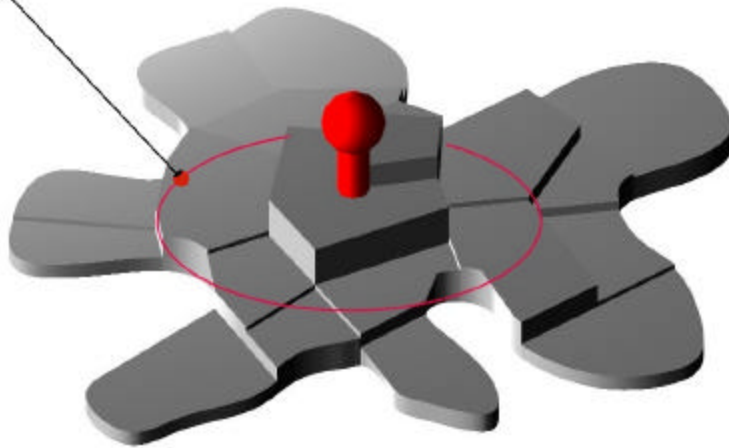


- Kain, John F. and William C. Apgar Jr. Simulation of Housing Market Dynamics. *AREUEA Journal*, 7(4), Winter 1979, pp. 505-538.
- Li, Mingche and H. James Brown. Micro-Neighborhood Externalities and Hedonic Housing Prices. *Land Economics*, 56, May 1980.
- Madden, Janice F. Urban Wage Gradients: Empirical Evidence. *Journal of Urban Economics*, 18, 1985, pp. 291-301.
- Malpezzi, Stephen. Urban Housing and Financial Markets: Some International Comparisons. *Urban Studies*, 27(6), December 1990, pp. 971-1022.
- Malpezzi, Stephen and Gwendolyn Ball. Measuring the Urban Policy Environment: An Exploratory Analysis Using Rent Controls. *Habitat International*, 17(2), 1993, pp. 39-52.
- Malpezzi, Stephen and Yeuh-Chuan Kung. The Flight to the Suburbs, Revisited: The Intraurban Distribution of Population and the Price of Housing. Paper presented to the American Real Estate and Urban Economics Association, New Orleans, January 1997.
- McDonald, John F. Econometric Studies of Urban Population Density: A Survey. *Journal of Urban Economics*, 26, 1989, pp. 361-85.
- Mieszkowski, Peter and Mahlon Straszheim (eds.). *Current Issues in Urban Economics*. Hopkins, 1979.
- Mills, Edwin S. *Studies in the Structure of the Urban Economy*. Johns Hopkins University Press, 1972.
- Mills, Edwin S. The Measurement and Determinants of Suburbanization. *Journal of Urban Economics*, 32, 1992, pp. 377-87.
- Mills, Edwin S. and Byong-Nak Song. *Urbanization and Urban Problems*. Harvard, 1979.
- Mills, Edwin S. and Jee Peng Tan. A Comparison of Urban Population Density Functions in Developed and Developing Countries. *Urban Studies*, 17(3), October 1980, pp. 313-21.
- Mills, Edwin S. and K. Ohta. Urbanization and Urban Problems. In H. Patrick and H. Rosovsky (eds.), *Asia's New Giant*. Brookings, 1976.
- Mills, Edwin S. and Richard Price. Metropolitan Suburbanization and Central City Problems. *Journal of Urban Economics*, 15, 1984, pp. 1-17.
- Mozolin, Mikhail. The Geography of Housing Values in the Transformation to a Market Economy - A Case Study of Moscow. *Urban Geography*, Vol. 15, No. 2, 1994, pp. 107-127.
- Muth, Richard F. *Cities and Housing*. University of Chicago Press, 1969.

- Muth, Richard F. Models of Land-Use, Housing, and Rent: An Evaluation. *Journal of Regional Science*, 25, 1985, pp. 593-606.
- Pirie, G.H. Measuring Accessibility: A Review and Proposal. *Environment and Planning A*, 11, 1979, pp. 299-312.
- Richardson, Harry W. Monocentric versus Polycentric Modes: The Future of Urban Economics in Regional Science. *Annals of Regional Science*, 22, 1988, pp. 1-12.
- Stewart, J. Suggested Principles of Social Physics. *Science*, 106, 1947, pp. 179-80.
- Straszheim, Mahlon R. The Theory of Urban Residential Structure. In Edwin S. Mills (ed.), *Handbook of Urban and Regional Economics*. Volume 2, North Holland, 1987.
- Turnbull, Geoffrey K. *Urban Consumer Theory*. American Real Estate and Urban Economics Association Monograph Series No. 2, 1995.
- Von Thunen, Johann H. *The Isolated State*. Original in German, pub. 1826, 1842, 1850; *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalolonomie*, 3rd ed. Jena: Gustav Fischer, 1930; Translated by Peter Hall, and by Bernard W. Dempsey.
- Wheaton, William C. Monocentric Models of Urban Land Use: Contributions and Criticisms. in Mieszkowski and Straszheim.
- World Bank. *Housing: Enabling Markets to Work*. World Bank Policy Paper, 1993 (a).
- World Bank. South Africa Urban Economic Mission Aide Memoire. Processed, 1993 (b).
- Yinger, John. City and Suburb: Urban Models with More Than One Employment Center. *Journal of Urban Economics*, 31, March 1992, pp. 181-205.

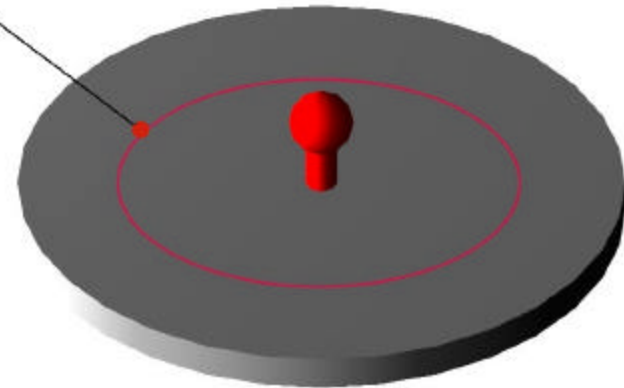
## Calculation of the Sprawl Index

Average distance per person to CBD = 3.41 km



3D representation of built up area and population densities  
Built up area = 100 km<sup>2</sup>

Average distance to center of circle = 3.76 km



Equivalent circle with an area of 100 km<sup>2</sup> and uniform density

$$\text{Sprawl Index} = \text{Average distance per person to CBD} / \text{Average distance to center of circle}$$
$$= 3.41 / 3.76 = 0.91$$

Figure 1

# Density Gradient and National GDP PC

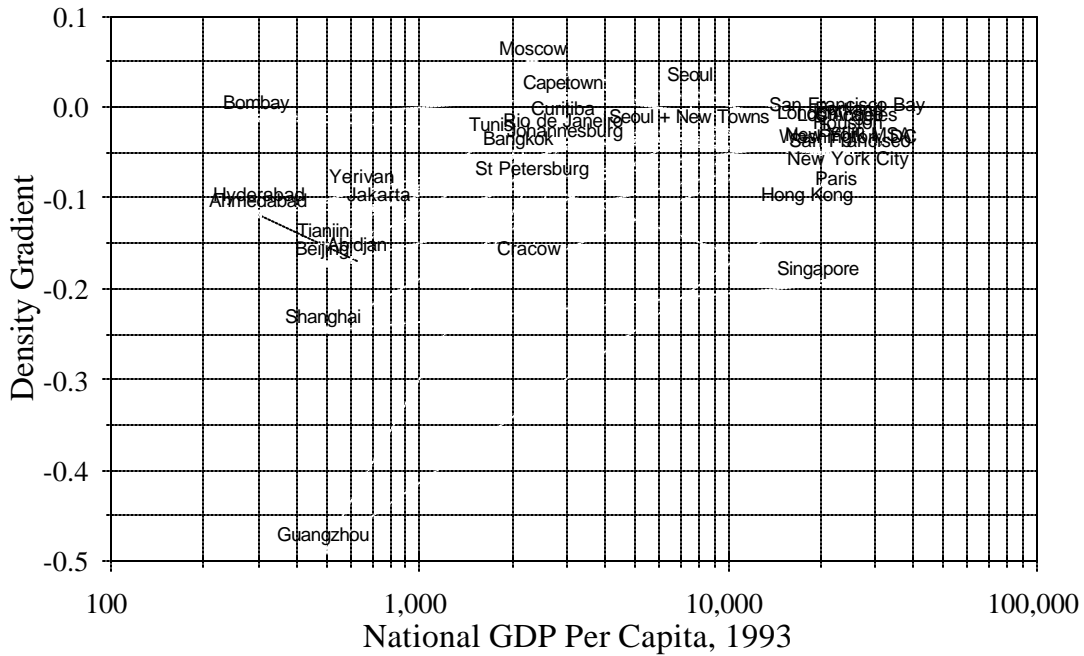


Figure 2

# Compactness and National GDP PC

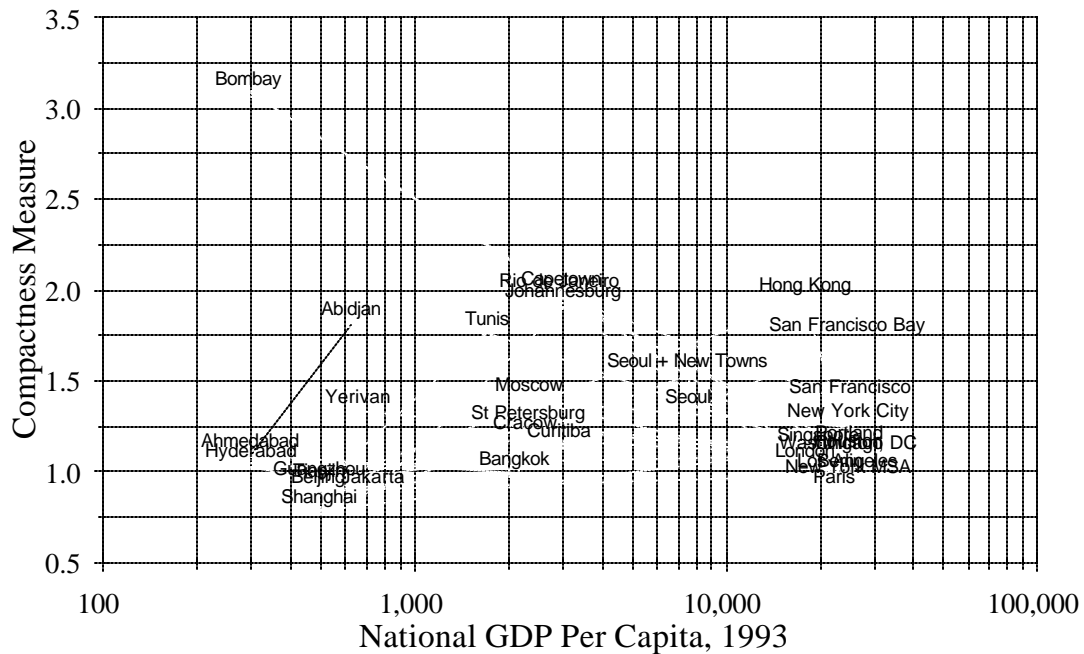
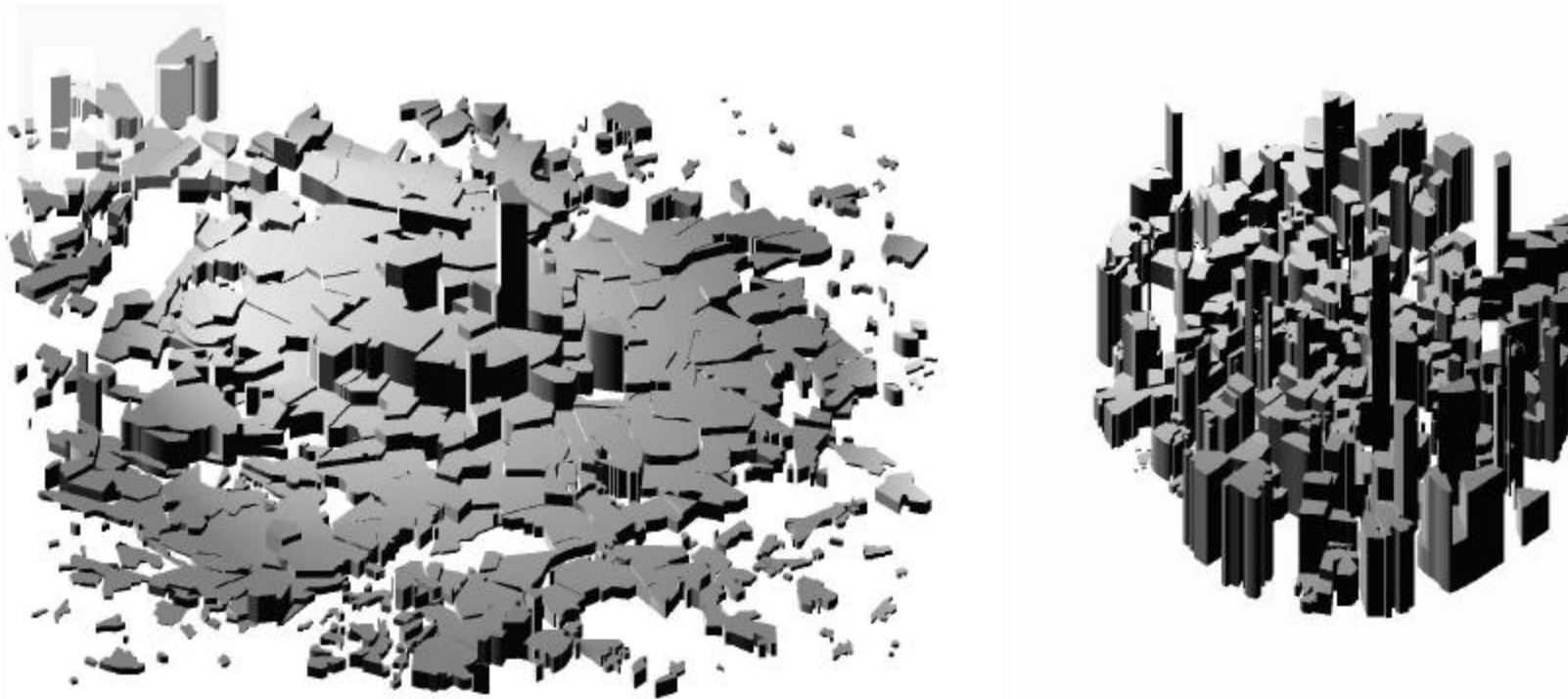


Figure 3



PARIS AND MOSCOW - 3D Representation of population densities in built-up areas



Source: Alain Bertaud

